A SOCRATIC DIALOGUE ON MATHEMATICS

SOCRATES Are you looking for somebody, my dear Hippocrates?

HIPPOCRATES No, Socrates, because I have already found him, namely you. I have been looking for you everywhere. Somebody told me at the agora that he saw you walking here along the River Ilissos; so I came after you.

SOCRATES Well then, tell me why you came, and then I want to ask you something about our discussion with Protagoras. Do you still remember it?

HIPPOCRATES How can you ask? Since that time not a single day has passed without my thinking about it. I came today to ask your advice because that discussion was on my mind.

SOCRATES It seems, my dear Hippocrates, that you want to talk to me about the very question I wish to discuss with you; thus the two subjects are one and the same. It seems that the mathematicians are mistaken in saying that two is never equal to one.

HIPPOCRATES As a matter of fact, Socrates, mathematics is just the topic I want to talk to you about.

SOCRATES Hippocrates, you certainly know that I am not a mathematician. Why did you not take your questions to the celebrated Theodoros?
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HIPPOCRATES You are amazing, Socrates, you answer my questions even before I tell you what they are. I came to ask your opinion about my becoming a pupil of Theodoros. When I came to you the last time, with the intention of becoming a pupil of Protagoras, we went to him together and you directed the discussion so that it became quite clear that he did not know the subject he taught. Thus I changed my mind and did not follow him. This discussion helped me to see what I should not do, but did not show me what I should do. I am still wondering about this. I visit banquets and the palaestra with young men of my age, I dare say I have a pleasant time, but this does not satisfy me. It disturbs me to feel myself ignorant. More precisely, I feel that the knowledge I have is rather uncertain. During the discussion with Protagoras, I realized that my knowledge about familiar notions like virtue, justice and courage was far from satisfactory. Nevertheless, I think it is great progress that I now see clearly my own ignorance.

SOCRATES I am glad, my dear Hippocrates, that you understand me so well. I always tell myself quite frankly that I know nothing. The difference between me and most other people is that I do not imagine I know what in reality I do not know.

HIPPOCRATES This clearly shows your wisdom, Socrates. But such knowledge is not enough for me. I have a strong desire to obtain some certain and solid knowledge, and I shall not be happy until I do. I am constantly pondering what kind of knowledge I should try to acquire. Recently, Theaitemes told me that certainty exists only in mathematics and suggested that I learn mathematics from his master, Theodoros, who is the leading expert on numbers and geometry in Athens. Now, I should not want to make the same mistake I made when I wanted to be a pupil of Protagoras. Therefore tell me, Socrates, shall I find the kind of sound knowledge I seek if I learn mathematics from Theodoros?
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Socrates If you want to study mathematics, O son of Apollodorus, then you certainly cannot do better than go to my highly esteemed friend Theodorus. But you must decide for yourself whether or not you really do want to study mathematics. Nobody can know your needs better than you yourself.

Hippocrates Why do you refuse to help me, Socrates? Perhaps I offended you without knowing it?

Socrates You misunderstand me, my young friend. I am not angry; but you ask the impossible of me. Everybody must decide for himself what he wants to do. I can do no more than assist as a midwife at the birth of your decision.

Hippocrates Please, my dear Socrates, do not refuse to help me, and if you are free now, let us start immediately.

Socrates Well, if you want to. Let us lie down in the shadow of that plane-tree and begin. But first tell me, are you ready to conduct the discussion in the manner I prefer? I shall ask the questions and you shall answer them. By this method you will come to see more clearly what you already know, for it brings into blossom the seeds of knowledge already in your soul. I hope you will not behave like King Darius who killed the master of his mines because he brought only copper out of a mine the king thought contained gold. I hope you do not forget that a miner can find in a mine only what it contains.

Hippocrates I swear that I shall make no reproaches, but, by Zeus, let us begin mining at once.

Socrates All right. Then tell me, do you know what mathematics is? I suppose you can define it since you want to study it.

Hippocrates I think every child could do so. Mathematics is one of the sciences, and one of the finest.

Socrates I did not ask you to praise mathematics, but to describe its nature. For instance, if I asked you about the art of physicians,
you would answer that this art deals with health and illness, and has the aim of healing the sick and preserving health. Am I right?

HIPPOCRATES Certainly.

SOCRATES Then answer me this: does the art of the physicians deal with something that exists or with something that does not exist? If there were no physicians, would illness still exist?

HIPPOCRATES Certainly, and even more than now.

SOCRATES Let us have a look at another art, say that of astronomy. Do you agree with me that astronomers study the motion of the stars?

HIPPOCRATES To be sure.

SOCRATES And if I ask you whether astronomy deals with something that exists, what is your answer?

HIPPOCRATES My answer is yes.

SOCRATES Would stars exist if there were no astronomers in the world?

HIPPOCRATES Of course. And if Zeus in his anger extinguished all mankind, the stars would still shine in the sky at night. But why do we discuss astronomy instead of mathematics?

SOCRATES Do not be impatient, my good friend. Let us consider a few other arts in order to compare them with mathematics. How would you describe the man who knows about all the creatures living in the woods or in the depths of the sea?

HIPPOCRATES He is a scientist studying living nature.

SOCRATES And do you agree that such a man studies things which exist?

HIPPOCRATES I agree.

SOCRATES And if I say that every art deals with something that exists, would you agree?
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HIPPOCRATES Completely.

SOCRATES Now tell me, my young friend, what is the object of mathematics? What things does a mathematician study?

HIPPOCRATES I have asked Theaitetos the same question. He answered that a mathematician studies numbers and geometrical forms.

SOCRATES Well, the answer is right, but would you say that these things exist?

HIPPOCRATES Of course. How can we speak of them if they do not exist?

SOCRATES Then tell me, if there were no mathematicians, would there be prime numbers, and if so, where would they be?

HIPPOCRATES I really do not know what to answer. Clearly, if mathematicians think about prime numbers, then they exist in their consciousness; but if there were no mathematicians, the prime numbers would not be anywhere.

SOCRATES Do you mean that we have to say mathematicians study non-existing things?

HIPPOCRATES Yes, I think we have to admit that.

SOCRATES Let us look at the question from another point of view. Here, I wrote on this wax tablet the number 37. Do you see it?

HIPPOCRATES Yes, I do.

SOCRATES And can you touch it with your hand?

HIPPOCRATES Certainly.

SOCRATES Then perhaps numbers do exist?

HIPPOCRATES O Socrates, you are mocking me. Look here, I have drawn on the same tablet a dragon with seven heads. Does it follow that such a dragon exists? I have never met anybody who has seen a dragon, and I am convinced that dragons do not exist at all except in fairy tales. But suppose I am mistaken, suppose
somewhere beyond the pillars of Heracles dragons really do exist, that still has nothing to do with my drawing.

Socrates You speak the truth, Hippocrates, and I agree with you completely. But does this mean that even though we can speak about them, and write them down, numbers nevertheless do not exist in reality?

Hippocrates Certainly.

Socrates Do not draw hasty conclusions. Let us make another trial. Am I right in saying that we can count the sheep here in the meadow or the ships in the harbor of Pireus?

Hippocrates Yes, we can.

Socrates And the sheep and the ships exist?

Hippocrates Clearly.

Socrates But if the sheep exist, their number must be something that exists, too?

Hippocrates You are making fun of me, Socrates. Mathematicians do not count sheep; that is the business of shepherds.

Socrates Do you mean, what mathematicians study is not the number of sheep or ships, or of other existing things, but the number itself? And thus they are concerned with something that exists only in their minds?

Hippocrates Yes, this is what I mean.

Socrates You told me that according to Theaitetos mathematicians study numbers and geometrical forms. How about forms? If I ask you whether they exist, what is your answer?

Hippocrates Certainly they exist. We can see the form of a beautiful vessel, for example, and feel it with our hands, too.

Socrates Yet I still have one difficulty. If you look at a vessel what do you see, the vessel or its form?
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HIPPOCRATES I see both.

SOCRATES Is that the same thing as looking at a lamb? Do you see the lamb and also its hair?

HIPPOCRATES I find the simile very well chosen.

SOCRATES Well, I think it limps like Hephaestus. You can cut the hair off the lamb and then you see the lamb without its hair, and the hair without the lamb. Can you separate in a similar way the form of a vessel from the vessel itself?

HIPPOCRATES Certainly not, and I dare say nobody can.

SOCRATES And nevertheless you still believe that you can see a geometric form?

HIPPOCRATES I am beginning to doubt it.

SOCRATES Besides this, if mathematicians study the forms of vessels, shouldn't we call them potters?

HIPPOCRATES Certainly.

SOCRATES Then if Theodoros is the best mathematician would he not be the best potter, too? I have heard many people praising him, but nobody has told me that he understands anything about pottery. I doubt whether he could make even the simplest pot. Or perhaps mathematicians deal with the form of statues or buildings?

HIPPOCRATES If they did, they would be sculptors and architects.

SOCRATES Well, my friend, we have come to the conclusion that mathematicians when studying geometry are not concerned with the forms of existing objects such as vessels, but with forms which exist only in their thoughts. Do you agree?

HIPPOCRATES I have to agree.

SOCRATES Having established that mathematicians are concerned with things that do not exist in reality, but only in their thoughts,
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let us examine the statement of Theaitetos, which you mentioned, that mathematics gives us more reliable and more trustworthy knowledge than does any other branch of science. Tell me, did Theaitetos give you some examples?

Hippocrates Yes, he said for instance that one cannot know exactly how far Athens is from Sparta. Of course, the people who travel that way agree on the number of days one has to walk, but it is impossible to know exactly how many feet the distance is. On the other hand, one can tell, by means of the theorem of Pythagoras, what the length of the diagonal of a square is. Theaitetos also said that it is impossible to give the exact number of people living in Hellas. If somebody tried to count all of them, he would never get the exact figure, because during the counting some old people would die and children would be born; thus the total number could be only approximately correct. But if you ask a mathematician how many edges a regular dodecahedron has, he will tell you that the dodecahedron is bounded by 12 faces, each having 5 edges. This makes 60, but as each edge belongs to two faces and thus has been counted twice, the number of edges of the dodecahedron is equal to 30, and this figure is beyond every doubt.

Socrates Did he mention any other examples?

Hippocrates Quite a few, but I do not remember all of them. He said that in reality you never find two things which are exactly the same. No two eggs are exactly the same, even the pillars of Poseidon’s temple are slightly different from each other; but one may be sure that the two diagonals of a rectangle are exactly equal. He quoted Heraclitus who said that everything which exists is constantly changing, and that sure knowledge is only possible about things which never change, for instance, the odd and the even, the straight line and the circle.

Socrates That will do. These examples convince me that in mathematics we can get knowledge which is beyond doubt, while
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in other sciences or in everyday life it is impossible. Let us try to summarize the results of our inquiry into the nature of mathematics. Am I right in saying we came to the conclusion that mathematics studies non-existing things and is able to find out the full truth about them?

HIPPOCRATES Yes, that is what we established.

SOCRATES But tell me, for Zeus's sake, my dear Hippocrates, is it not mysterious that one can know more about things which do not exist than about things which do exist?

HIPPOCRATES If you put it like that, it certainly is a mystery. I am sure there is some mistake in our arguments.

SOCRATES No, we proceeded with the utmost care and we controlled every step of the argument. There cannot be any mistake in our reasoning. But listen, I remember something which may help us to solve the riddle.

HIPPOCRATES Tell me quickly, because I am quite bewildered.

SOCRATES This morning I was in the hall of the second archon, where the wife of a carpenter from the village Pithos was accused of betraying and, with the aid of her lover, murdering her husband. The woman protested and swore to Artemis and Aphrodite that she was innocent, that she never loved anyone but her husband, and that her husband was killed by pirates. Many people were called as witnesses. Some said that the woman was guilty, others said that she was innocent. It was impossible to find out what really happened.

HIPPOCRATES Are you mocking me again? First you confused me completely, and now instead of helping me to find the truth you tell me such stories.

SOCRATES Do not be angry, my friend, I have serious reasons for speaking about this woman whose guilt it was impossible to ascertain. But one thing is sure. The woman exists. I saw her with my own eyes, and of anyone who was there, many of whom
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have never lied in their lives, you can ask the same question and you will receive the same answer.

HIPPOCRATES Your testimony is sufficient for me, my dear Socrates. Let it be granted that the woman exists. But what has this fact to do with mathematics?

SOCRATES More than you imagine. But tell me first, do you know the story about Agamemnon and Clytemnestra?

HIPPOCRATES Everybody knows the story. I saw the trilogy of Aeschylus at the theatre last year.

SOCRATES Then tell me the story in a few words.

HIPPOCRATES While Agamemnon, the king of Mycenae, fought under the walls of Troy, his wife, Clytemnestra, committed adultery with Aegisthus, the cousin of her husband. After the fall of Troy, when Agamemnon returned home, his wife and her lover murdered him.

SOCRATES Tell me Hippocrates, is it quite sure that Clytemnestra was guilty?

HIPPOCRATES I do not understand why you ask me such questions. There can be no doubt about the story. According to Homer, when Odysseus visited the underworld he met Agamemnon, who told Odysseus his sad fate.

SOCRATES But are you sure that Clytemnestra and Agamemnon and all the other characters of the story really existed?

HIPPOCRATES Perhaps I would be ostracized if I said this in public, but my opinion is that it is impossible either to prove or disprove today, after so many centuries, whether the stories of Homer are true or not. But this is quite irrelevant. When I told you that Clytemnestra was guilty, I did not speak about the real Clytemnestra—if such a person ever lived—but about the Clytemnestra of our Homeric tradition, about the Clytemnestra in the trilogy of Aeschylus.
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Socrates May I say that we know nothing about the real Clytemnestra? Even her existence is uncertain, but as regards the Clytemnestra who is a character in the trilogy of Aeschylus, we are sure that she was guilty and murdered Agamemnon because that is what Aeschylus tells us.

Hippocrates Yes, of course. But why do you insist on all this?

Socrates You will see in a moment. Let me summarize what we found out. It is impossible in the case of the flesh and blood woman who was tried today in Athens to establish whether she is guilty, while there can be no doubt about the guilt of Clytemnestra who is a character in a play and who probably never existed. Do you agree?

Hippocrates Now I am beginning to understand what you want to say. But it would be better if you drew the conclusions yourself.

Socrates The conclusion is this: we have much more certain knowledge about persons who exist only in our imagination, for example about characters in a play, than about living persons. If we say that Clytemnestra was guilty, it means only that this is how Aeschylus imagined her and presented her in his play. The situation is exactly the same in mathematics. We may be sure that the diagonals of a rectangle are equal because this follows from the definition of a rectangle given by mathematicians.

Hippocrates Do you mean, Socrates, that our paradoxical result is really true and one can have a much more certain knowledge about non-existent things—for instance about the objects of mathematics—than about the real objects of nature? I think that now I also see the reason for this. The notions which we ourselves have created are by their very nature completely known to us, and we can find out the full truth about them because they have no other reality outside our imagination. However, the objects
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which exist in the real world are not identical with our picture of them, which is always incomplete and approximate; therefore our knowledge about these real things can never be complete or quite certain.

Socrates That is the truth, my young friend, and you stated it better than I could have.

Hippocrates This is to your credit, Socrates, because you led me to understand these things. I see now not only that Theaïtétos was quite right in telling me I must study mathematics if I want to obtain unfailing knowledge, but also why he was right. However, if you have guided me with patience up to now, please do not abandon me yet because one of my questions, in fact the most important one, is still unanswered.

Socrates What is this question?

Hippocrates Please remember, Socrates, that I came to ask your advice as to whether I should study mathematics. You helped me to realize that mathematics and only mathematics can give me the sort of sound knowledge I want. But what is the use of this knowledge? It is clear that if one obtains some knowledge about the existing world, even if this knowledge is incomplete and is not quite certain, it is nevertheless of value to the individual as well as to the state. Even if one gets some knowledge about things such as the stars, it may be useful, for instance in navigation at night. But what is the use of knowledge of non-existing things such as that which mathematics offers? Even if it is complete and beyond any doubt, what is the use of knowledge concerning things which do not exist in reality?

Socrates My dear friend, I am quite sure you know the answer, only you want to examine me.

Hippocrates By Heracles, I do not know the answer. Please help me.
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SOCRATES Well, let us try to find it. We have established that the notions of mathematics are created by the mathematician himself. Tell me, does this mean that the mathematician chooses his notions quite arbitrarily as it pleases him?

HIPPOCRATES As I told you, I do not yet know much about mathematics. But it seems to me that the mathematician is as free to choose the objects of his study as the poet is free to choose the characters of his play, and as the poet invests his characters with whatever traits please him, so can the mathematician endow his notions with such properties as he likes.

SOCRATES If this were so, there would be as many mathematical truths as there are mathematicians. How do you explain, then, that all mathematicians study the same notions and problems? How do you explain that, as often happens, mathematicians living far from each other and having no contact independently discover the same truths? I never heard of two poets writing the same poem.

HIPPOCRATES Nor have I heard of such a thing. But I remember Theaitetos telling me about a very interesting theorem he discovered on incommensurable distances. He showed his results to his master, Theodoros, who produced a letter by Archytas in which the same theorem was contained almost word for word.

SOCRATES In poetry that would be impossible. Now you see that there is a problem. But let us continue. How do you explain that the mathematicians of different countries can usually agree about the truth, while about questions concerning the state, for example, the Persians and the Spartans have quite opposite views from ours in Athens, and, moreover, we here do not often agree with each other?

HIPPOCRATES I can answer that last question. In matters concerning the state everybody is personally interested, and these personal interests are often in contradiction. This is why it is difficult to
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come to an agreement. However, the mathematician is led purely by his desire to find the truth.

SOCRATES Do you mean to say that the mathematicians are trying to find a truth which is completely independent of their own person?

HIPPOCRATES Yes, I do.

SOCRATES But then we were mistaken in thinking that mathematicians choose the objects of their study at their own will. It seems that the object of their study has some sort of existence which is independent of their person. We have to solve this new riddle.

HIPPOCRATES I do not see how to start.

SOCRATES If you still have patience, let us try it together. Tell me, what is the difference between the sailor who finds an uninhabited island and the painter who finds a new color, one which no other painter has used before him?

HIPPOCRATES I think that the sailor may be called a discoverer, and the painter an inventor. The sailor discovers an island which existed before him, only it was unknown, while the painter invents a new color which before that did not exist at all.

SOCRATES Nobody could answer the question better. But tell me, the mathematician who finds a new truth, does he discover it or invent it? Is he a discoverer as the sailor or an inventor as the painter?

HIPPOCRATES It seems to me that the mathematician is more like a discoverer. He is a bold sailor who sails on the unknown sea of thought and explores its coasts, islands and whirlpools.

SOCRATES Well said, and I agree with you completely. I would add only that to a lesser extent the mathematician is an inventor too, especially when he invents new concepts. But every dis-
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coverer has to be, to a certain extent, an inventor too. For in-
stance, if a sailor wants to get to places which other sailors before
him were unable to reach, he has to build a ship that is better
than the ships other sailors used. The new concepts invented
by the mathematicians are like new ships which carry the dis-
coverer farther on the great sea of thought.

Hippocrates My dear Socrates, you helped me to find the an-
swer to the question which seemed so difficult to me. The main
aim of the mathematician is to explore the secrets and riddles
of the sea of human thought. These exist independently of the
person of the mathematician, though not from humanity as a
whole. The mathematician has a certain freedom to invent new
concepts as tools, and it seems that he could do this at his dis-
cretion. However, he is not quite free in doing this because the
new concepts have to be useful for his work. The sailor also can
build any sort of ship at his discretion, but, of course, he would
be mad to build a ship which would be crushed to pieces by the
first storm. Now I think that everything is clear.

Socrates If you see everything clearly, try again to answer the
question: what is the object of mathematics?

Hippocrates We came to the conclusion that besides the world
in which we live, there exists another world, the world of human
thought, and the mathematician is the fearless sailor who ex-
plores this world, not shrinking back from the troubles, dangers
and adventures which await him.

Socrates My friend, your youthful vigor almost sweeps me off
my feet, but I am afraid that in the ardor of your enthusiasm you
overlook certain questions.

Hippocrates What are these questions?

Socrates I do not want to disappoint you, but I feel that your
main question has not yet been answered. We have not yet an-
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answered the question: what is the use of exploring the wonderful sea of human thought?

HIPPOCRATES You are right, my dear Socrates, as always. But won't you put aside your method this time and tell me the answer immediately?

SOCRATES No, my friend, even if I could, I would not do this, and it is for your sake. The knowledge somebody gets without work is almost worthless to him. We understand thoroughly only that which—perhaps with some outside help—we find out ourselves, just as a plant can use only the water which it sucks up from the soil through its own roots.

HIPPOCRATES All right, let us continue our search by the same method, but at least help me by a question.

SOCRATES Let us go back to the point where we established that the mathematician is not dealing with the number of sheep, ships or other existing things, but with the numbers themselves. Don't you think, however, that what the mathematicians discover to be true for pure numbers is true for the number of existing things too? For instance, the mathematician finds that 17 is a prime number. Therefore, is it not true that you cannot distribute 17 living sheep to a group of people, giving each the same number, unless there are 17 people?

HIPPOCRATES Of course, it is true.

SOCRATES Well, how about geometry? Can it not be applied in building houses, in making pots or in computing the amount of grain a ship can hold?

HIPPOCRATES Of course, it can be applied, though it seems to me that for these practical purposes of the craftsman not too much mathematics is needed. The simple rules known already by the clerks of the pharaohs in Egypt are sufficient for most such purposes, and the new discoveries about which Theaitetos spoke to
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me with such overflowing fervor are neither used nor needed in practice.

SOCRATES Perhaps not at the moment, but they may be used in the future.

HIPPOCRATES I am interested in the present.

SOCRATES If you want to be a mathematician, you must realize you will be working mostly for the future. Now, let us return to the main question. We saw that knowledge about another world of thought, about things which do not exist in the usual sense of the word, can be used in everyday life to answer questions about the real world. Is this not surprising?

HIPPOCRATES More than that, it is incomprehensible. It is really a miracle.

SOCRATES Perhaps it is not so mysterious at all, and if we open the shell of this question, we may find a real pearl.

HIPPOCRATES Please, my dear Socrates, do not speak in puzzles like the Pythia.

SOCRATES Tell me then, are you surprised when somebody who has travelled in distant countries, who has seen and experienced many things, returns to his city and uses his experience to give good advice to his fellow citizens?

HIPPOCRATES Not at all.

SOCRATES Even if the countries which the traveller has visited are very far away and are inhabited by quite a different sort of people, speaking another language, worshipping other gods?

HIPPOCRATES Not even in that case, because there is much that is common between different people.

SOCRATES Now tell me, if it turned out that the world of mathematics is, in spite of its peculiarities, in some sense similar to our
real world, would you still find it miraculous that mathematics can be applied to the study of the real world?

HIPPOCRATES In that case no, but I do not see any similarity between the real world and the imaginary world of mathematics.

SOCRATES Do you see that rock on the other side of the river, there where the river broadens out and forms a lake?

HIPPOCRATES I see it.

SOCRATES And do you see the image of the rock reflected in the water?

HIPPOCRATES Certainly I do.

SOCRATES Then tell me, what is the difference between the rock and its reflection?

HIPPOCRATES The rock is a solid piece of hard matter. It is made warm by the sun. If you touched it, you would feel that it is rough. The reflected image cannot be touched; if I put my hand on it, I would touch only the cool water. As a matter of fact, the reflected image does not really exist; it is illusion, nothing else.

SOCRATES Is there nothing in common between the rock and its reflected image?

HIPPOCRATES Well, in a certain sense, the reflected image is a faithful picture of the rock. The contour of the rock, even its small abutments, are clearly visible in the reflected image. But what of it? Do you want to say that the world of mathematics is a reflected image of the real world in the mirror of our thinking?

SOCRATES You said it, and very well.

HIPPOCRATES But how is that possible?

SOCRATES Let us recall how the abstract concepts of mathematics developed. We said that the mathematician deals with pure
numbers, and not with the numbers of real objects. But do you think that somebody who has never counted real objects can understand the abstract notion of number? When a child learns counting, he first counts pebbles and small sticks. Only if he knows that two pebbles and three pebbles make five pebbles, and the same about sticks or coins, is he able to understand that two and three make five. The situation is essentially the same with geometry. The child arrives at the notion of a sphere through experiences with round objects like balls. Mankind developed all fundamental notions of mathematics in a similar way. These notions are crystallized from a knowledge of the real world, and thus it is not surprising but quite natural that they bear the marks of their origin, as children do of their parents. And exactly as children when they grow up become the supporters of their parents, so any branch of mathematics, if it is sufficiently developed, becomes a useful tool in exploring the real world.

HIPPOCRATES Now it is quite clear to me how a knowledge of the non-existent things of the world of mathematics can be used in everyday life. You rendered me a great service in helping me to understand this.

SOCRATES I envy you, my dear Hippocrates, because I still wonder about one thing which I should like to have settled. Perhaps you can help me.

HIPPOCRATES I would do so with pleasure, but I am afraid you are mocking me again. Do not make me ashamed by asking my help, but tell me frankly the question which I overlooked.

SOCRATES You will see it yourself if you try to summarize the results of our discussion.

HIPPOCRATES Well, when it became clear why mathematics is able to give certain knowledge about a world different from
the world in which we live, about the world of human thought, the question remained as to the use of this knowledge. Now we have found that the world of mathematics is nothing else but a reflection in our mind of the real world. This makes it clear that every discovery about the world of mathematics gives us some information about the real world. I am completely satisfied with this answer.

Socrates If I tell you the answer is not yet complete, I do so not because I want to confuse you, but because I am sure that sooner or later you will raise the question yourself and will reproach me for not having called your attention to it. You would say: “Tell me, Socrates, what is the sense of studying the reflected image if we can study the object itself?”

Hippocrates You are perfectly right; it is an obvious question. You are a wizard, Socrates. You can totally confuse me by a few words, and you can knock down by an innocent-looking question the whole edifice which we have built with so much trouble. I should, of course, answer that if we are able to have a look at the original thing, it makes no sense to look at the reflected image. But I am sure this shows only that our simile fails at this point. Certainly there is an answer, only I do not know how to find it.

Socrates Your guess is correct that the paradox arose because we kept too close to the simile of the reflected image. A simile is like a bow—if you stretch it too far, it snaps. Let us drop it and choose another one. You certainly know that travellers and sailors make good use of maps.

Hippocrates I have experienced that myself. Do you mean that mathematics furnishes a map of the real world?

Socrates Yes. Can you now answer the question: what advantage would it be to look at the map instead of looking at the landscape?
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HIPPOCRATES This is clear: using the map we can scan vast distances which could be covered only by travelling many weeks or months. The map shows us not every detail, but only the most important things. Therefore it is useful if we want to plan a long voyage.

SOCRATES Very well. But there is something else which occurred to me.

HIPPOCRATES What is it?

SOCRATES There is another reason why the study of the mathematical image of the world may be of use. If mathematicians discover some property of the circle, this at once gives us some information about any object of circular shape. Thus, the method of mathematics enables us to deal with different things at the same time.

HIPPOCRATES What about the following similes: If somebody looks at a city from the top of a nearby mountain, he gets a more comprehensive view than if he walks through its crooked streets; or if a general watches the movements of an enemy army from a hill, he gets a clearer picture of the situation than does the soldier in the front line who sees only those directly opposite him.

SOCRATES Well, you surpass me in inventing new similes, but as I do not want to fall behind, let me also add one parable. Recently I looked at a painting by Aristophon, the son of Aglaophon, and the painter warned me, "If you go too near the picture, Socrates, you will see only colored spots, but you will not see the whole picture."

HIPPOCRATES Of course, he was right, and so were you, when you did not let us finish our discussion before we got to the heart of the question. But I think it is time for us to return to the city because the shadows of night are falling and I am hungry and
thirsty. If you still have some patience, I would like to ask you something while we walk.

Socrates All right, let us start and you may ask your question.

Hippocrates Our discourse convinced me fully that I should start studying mathematics and I am very grateful to you for this. But tell me, why are you yourself not doing mathematics? Judging from your deep understanding of the real nature and importance of mathematics, it is my guess that you would surpass all other mathematicians of Hellas, were you to concentrate on it. I would be glad to follow you as your pupil in mathematics if you accepted me.

Socrates No, my dear Hippocrates, this is not my business. Theodoros knows much more about mathematics than I do and you cannot find a better master than him. As to your question of why I myself am not a mathematician, I shall give you the reasons. I do not conceal my high opinion about mathematics. I think that we Hellenes have in no other art made such important progress as in mathematics, and this is only the beginning. If we do not extinguish each other in mad wars, we shall obtain wonderful results as discoverers as well as inventors. You asked me why I do not join the ranks of those who develop this great science. As a matter of fact, I am some sort of a mathematician, only of a different kind. An inner voice, you may call it an oracle, to which I always listen carefully, asked me many years ago, “What is the source of the great advances which the mathematicians have made in their noble science?” I answered, “I think the source of the success of mathematicians lies in their methods, the high standards of their logic, their striving without the least compromise to the full truth, their habit of starting always from first principles, of defining every notion used exactly and of avoiding self-contradictions.” My inner voice answered, “Very well, but why do you think, Socrates, that this method of think-
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...ing and arguing can be used only for the study of numbers and geometric forms? Why do you not try to convince your fellow citizens to apply the same high logical standards in every other field, for instance in philosophy and politics, in discussing the problems of everyday private and public life?” From that time on, this has been my goal. I have demonstrated (you remember, for instance, our discussion with Protagoras) that those who are thought to be wise men are mostly ignorant fools. All their arguing lacks solid foundation, since they use—contrary to mathematicians—undefined and only half-understood notions. By this activity I have succeeded in making almost everybody my enemy. This is not surprising because for all people who are sluggish in thinking and idly content to use obscure terms, I am a living reproach. People do not like those who constantly remind them of the faults which they are unable or unwilling to correct. The day will come when these people will fall upon me and exterminate me. But until that day comes, I shall continue to follow my calling. You, however, go to Theodoros.